Appendix to "Bisourcing in the Global Economy"

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Derivation of (6)

As M chooses an amount of m to maximize $(1-\beta_k)R - wm$ and H provides an amount of headquarters service h to maximize $\beta_k R - wh$, the first order conditions for the profit maximization problems are

$$\begin{cases} \alpha(1-\beta_k)\mu IY^{-\alpha}\theta^{\alpha}[\frac{h}{\eta}]^{\alpha\eta}[\frac{m}{1-\eta}]^{\alpha(1-\eta)-1} = w \\ \alpha\beta_k\mu IY^{-\alpha}\theta^{\alpha}[\frac{h}{\eta}]^{\alpha\eta-1}[\frac{m}{1-\eta}]^{\alpha(1-\eta)} = w \end{cases}$$

which yields the equilibrium value of the two inputs as

$$\begin{cases} h_k = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(\beta_k)^{\frac{\alpha}{1-\alpha}\eta+1}(1-\beta_k)^{\frac{\alpha}{1-\alpha}(1-\eta)}w^{-\frac{\alpha}{1-\alpha}} \\ m_k = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(\beta_k)^{\frac{\alpha}{1-\alpha}\eta}(1-\beta_k)^{\frac{\alpha}{1-\alpha}(1-\eta)+1}w^{-\frac{\alpha}{1-\alpha}} \end{cases}$$

The total profits are calculated as

$$\begin{aligned} \pi_k &= R - wh_k - wm_k - wf_k \\ &= \alpha^{\frac{\alpha}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(\beta_k)^{\frac{\alpha}{1-\alpha}\eta}(1-\beta_k)^{\frac{\alpha}{1-\alpha}(1-\eta)}w^{-\frac{\alpha}{1-\alpha}}[1-\alpha(\beta_k\eta \\ &\quad +(1-\beta_k)(1-\eta))] - wf_k \\ &= \phi_k(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}} - wf_k \end{aligned}$$

which gives Equation (6).

Derivation of (10)

The first order conditions for the headquarters (H), the internal supplier

(M_1) and the external supplier (M_2) are

$$\begin{cases} w = & \alpha\mu I Y^{-\alpha}\theta^{\alpha}\left[\frac{h}{\eta}\right]^{\alpha\eta-1} \\ & \left\{\beta_O\beta_I\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)} + (1-\beta_O)\beta_I\left[\frac{m_1}{1-\eta}\right]^{\alpha(1-\eta)} + \beta_O(1-\beta_I)\left[\frac{m_2}{1-\eta}\right]^{\alpha(1-\eta)}\right\} \\ w = & \alpha(1-\beta_I)\mu I Y^{-\alpha}\theta^{\alpha}\left[\frac{h}{\eta}\right]^{\alpha\eta} \\ & \left\{\beta_O\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} + (1-\beta_O)\left[\frac{m_1}{1-\eta}\right]^{\alpha(1-\eta)-1}\right\} \\ w = & \alpha(1-\beta_O)\mu I Y^{-\alpha}\theta^{\alpha}\left[\frac{h}{\eta}\right]^{\alpha\eta}\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} \end{cases} \tag{A.1.}$$

Dividing the second equation by the last one, we get

$$\begin{aligned} (1-\beta_O)\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} &= (1-\beta_I)\beta_O\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} \\ &\quad +(1-\beta_O)(1-\beta_I)\left[\frac{m_1}{1-\eta}\right]^{\alpha(1-\eta)-1} \end{aligned}$$

$\Rightarrow$

$$\frac{m_1+m_2}{m_1} = \left[\frac{1+\beta_O\beta_I-2\beta_O}{(1-\beta_O)(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}} \equiv A > 1 \tag{A.2.}$$

and $m_2 = (A-1)m_1$.

Substitute (A.2.) back into (A.1.), we get

$$\begin{cases} w = & \alpha\mu I Y^{-\alpha}\theta^{\alpha}\left[\frac{h}{\eta}\right]^{\alpha\eta-1}\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)}[\beta_O\beta_I + (1-\beta_O)\beta_I A^{-\alpha(1-\eta)} \\ & +\beta_O(1-\beta_I)\left(\frac{A-1}{A}\right)^{\alpha(1-\eta)}] \\ w = & \alpha(1-\beta_O)\mu I Y^{-\alpha}\theta^{\alpha}\left[\frac{h}{\eta}\right]^{\alpha\eta}\left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} \end{cases} \tag{A.3.}$$

Divide the second equation by the first one, we can get

$$\frac{\frac{h}{\eta}}{\frac{m_1+m_2}{1-\eta}} = \frac{T}{S}$$

where

$$\begin{aligned} T &= \beta_O\beta_I + (1-\beta_O)\beta_I A^{-\alpha(1-\eta)} + \beta_O(1-\beta_I)\left(\frac{A-1}{A}\right)^{\alpha(1-\eta)} \\ S &= 1-\beta_O \end{aligned}$$

Denote $\frac{h}{\eta} = Ta$ and $\frac{m_1+m_2}{1-\eta} = Sa$ and substitute into (A.3.), we can derive

$$a = \alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} T^{\frac{\alpha}{1-\alpha}\eta} S^{\frac{\alpha}{1-\alpha}(1-\eta)} w^{\frac{-1}{1-\alpha}}$$

$\Rightarrow$

$$\begin{cases} h_B = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} T^{\frac{\alpha}{1-\alpha}\eta+1} S^{\frac{\alpha}{1-\alpha}(1-\eta)} w^{\frac{-1}{1-\alpha}} \\ m_{1B} = \frac{(1-\eta)}{A}\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} T^{\frac{\alpha}{1-\alpha}\eta} S^{\frac{\alpha}{1-\alpha}(1-\eta)+1} w^{\frac{-1}{1-\alpha}} \\ m_{2B} = \frac{(1-\eta)(A-1)}{A}\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} T^{\frac{\alpha}{1-\alpha}\eta} S^{\frac{\alpha}{1-\alpha}(1-\eta)+1} w^{\frac{-1}{1-\alpha}} \end{cases}$$

Thus we can derive the profit as

$$\begin{aligned} \pi_B &= R - wh_B - wm_{1B} - wm_{2B} - wf_B \\ &= \alpha^{\frac{\alpha}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} T^{\frac{\alpha}{1-\alpha}\eta}(1-\beta_O)^{\frac{\alpha}{1-\alpha}(1-\eta)} w^{\frac{-\alpha}{1-\alpha}}[1 - \\ &\quad \alpha(\eta T + (1-\eta)(1-\beta_O))] - wf_B \\ &= \phi_B(\mu I)^{\frac{1}{1-\alpha}} Y^{-\frac{\alpha}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} - wf_B \end{aligned}$$

which gives Equation (10).

Proof of Lemma 1

When H negotiates with M_2 before M_1, by the symmetry of the bargaining game, we can write out the profit functions for the three parties (H, M_1 and M_2) as

$$\begin{cases} \pi_h = R - P_1 - P_2 = \beta_O\beta_I R + \beta_I(1-\beta_O)R_1 + \beta_O(1-\beta_I)R_2 - wh \\ \pi_1 = P_1 - wm_1 = (1-\beta_I)(R-R_2) - wm_1 \\ \pi_2 = P_2 - wm_2 = (1-\beta_O)[\beta_I R + (1-\beta_I)R_2 - \beta_I R_1] - wm_2 \end{cases}$$

The first order conditions of the suppliers are derived as

$$\begin{aligned} \frac{\partial \pi_1}{\partial m_1} &= \alpha(1-\beta_I)\mu I Y^{-\alpha}\theta^\alpha \left[\frac{h}{\eta}\right]^{\alpha\eta} \left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} - w \\ \frac{\partial \pi_2}{\partial m_2} &= \alpha(1-\beta_O)\mu I Y^{-\alpha}\theta^\alpha \left[\frac{h}{\eta}\right]^{\alpha\eta} \{\beta_I \left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} + \\ &\quad (1-\beta_I)\left[\frac{m_2}{1-\eta}\right]^{\alpha(1-\eta)-1}\} - w \end{aligned}$$

First, we have $\beta_I \left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} + (1-\beta_I)\left[\frac{m_2}{1-\eta}\right]^{\alpha(1-\eta)-1} > \left[\frac{m_1+m_2}{1-\eta}\right]^{\alpha(1-\eta)-1}$, that is, for the last unit of investment, the external supplier's contribution to the total revenue is higher than the internal supplier's.

Second, the external supplier has a larger Nash bargaining power than the internal supplier does, i.e. $1-\beta_O > 1-\beta_I$, which means the external supplier may claim a larger share of her contribution to the total revenue than the internal supplier does. Based on these two results, we conclude that $\frac{\partial \pi_2}{\partial m_2} > \frac{\partial \pi_1}{\partial m_1}$.

As the external supplier maximizes her profits, she will make investments m_2 up to the point of $\frac{\partial \pi_2}{\partial m_2} = 0$. As a result, at the optimum, we have

$$\frac{\partial \pi_2}{\partial m_2} = 0 > \frac{\partial \pi_1}{\partial m_1}$$

Given that M_1 makes investments m_1 after M_2 does and M_2 invests m_2 up to a level corresponding to $\frac{\partial \pi_2}{\partial m_2} = 0$, any investment made by M_1 will have a negative marginal profit. As a consequence, the internal supplier will have no incentive to make any investment as its share of the total revenue cannot cover its costs of investments.

Proof of Lemma 2

The optimal reaction curves of the heaquarters and the supplier are illustrated in Figure 1. The optimal investment levels are thus determined by the interaction of these two reaction curves under each organization form.

Based on the findings of the property rights theory of the firm, it is known that the optimal reaction curve of the supplier under single insourcing is to the left of that under single outsourcing while the optimal reaction curve of the headquarters under single insourcing is above that under single outsourcing. Meanwhile, since under the bisourcing strategy, the headquarters will optimally choose to bargain with the internal supplier first and then with the external supplier, the external supplier is the final maker of the component production. Thus the optimal reaction curve of the supplier under bisourcing is the same to that under single outsourcing. It then remains to compare the optimal reaction curve of the headquarter under bisourcing with that under single insourcing and single outsourcing.

The slope of the headquarters optimal reaction curve under single insourcing, single outsourcing and bisourcing are represented by β_I, β_O and T, respectively. We show in the following that when β_I is not sufficiently large, $T \geq \beta_I$.

$$T = \beta_O\beta_I + (1-\beta_O)\beta_I \left(\frac{1}{A}\right)^{\alpha(1-\eta)} + \beta_O(1-\beta_I)\left(1-\frac{1}{A}\right)^{\alpha(1-\eta)} \gtreqless \beta_I$$

$\Leftrightarrow$

$$\beta_O(1-\beta_I)\left(1-\frac{1}{A}\right)^{\alpha(1-\eta)} \gtreqless \beta_I(1-\beta_O)\left[1-\left(\frac{1}{A}\right)^{\alpha(1-\eta)}\right]$$

$\Leftrightarrow$

$$\frac{\beta_O(1-\beta_I)}{\beta_I(1-\beta_O)} \gtreqless \frac{1-\left(\frac{1}{A}\right)^{\alpha(1-\eta)}}{\left(1-\frac{1}{A}\right)^{\alpha(1-\eta)}} = \frac{A^{\alpha(1-\eta)}-1}{(A-1)^{\alpha(1-\eta)}}$$

where $A = \left[\frac{1+\beta_O\beta_I-2\beta_O}{(1-\beta_O)(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}} \geq 1$.

Let $L(\beta_I;\beta_O) = \frac{\beta_O(1-\beta_I)}{\beta_I(1-\beta_O)}$ and $R(\beta_I;\beta_O,\alpha,\eta) = \frac{A^{\alpha(1-\eta)}-1}{(A-1)^{\alpha(1-\eta)}}$. It is easy to show that L is a monotonically decreasing and concave function of β_I, i.e. $\frac{\partial L}{\partial \beta_I} = -\frac{L}{(1-\beta_I)\beta_I} < 0$ and $\frac{\partial^2 L}{\partial \beta_I^2} = 2L\left[\frac{1}{\beta_I^2}+\frac{1}{(1-\beta_I)\beta_I}\right] > 0$. And $\frac{\partial R}{\partial \beta_I} = R\alpha(1-\eta)\frac{\beta_I-\beta_O}{(A^{\alpha(1-\eta)}-1)(A-1)(1+\beta_O\beta_I-2\beta_O)}\frac{\partial A}{\partial \beta_I} > 0$ as $\frac{\partial A}{\partial \beta_I} = \frac{A}{1-\alpha(1-\eta)}\left[\frac{\beta_O}{1+\beta_O\beta_I-2\beta_O}+\frac{1}{1-\beta_I}\right] > 0$. Moreover, $L(\beta_I \to \beta_O) \to 1 > R(\beta_I \to \beta_O) \to 0$ and $L(\beta_I \to 1) \to 0 < R(\beta_I \to 1) \to 1$. So there exists a β_I^* when $\beta_I \leq \beta_I^*$, $L(\beta_I;\beta_O) \geq R(\beta_I;\beta_O,\alpha,\eta)$, which equals to $T \geq \beta_I$; and when $\beta_I > \beta_I^*$, $L(\beta_I;\beta_O) < R(\beta_I;\beta_O,\alpha,\eta)$, which equals to $T < \beta_I$.

Thus when β_I is not so large, the optimal reaction curve of the headquarters are highest under bisourcing, followed by single insourcing and single outsourcing.

With these results in hand, it is easy to show from Figure 1 that bisourcing has the higher investment levels than both single insourcing and single outsourcing.

Derivation of $\boldsymbol{\beta_I^*}$

$$T(x) = \beta_O\beta_I + (1-\beta_O)\beta_I x^{\alpha(1-\eta)} + \beta_O(1-\beta_I)(1-x)^{\alpha(1-\eta)}$$

Differentiating $T(x)$ with respect to x, we get

$$\begin{aligned} \frac{\partial T}{\partial x} &= \alpha(1-\eta)\left[(1-\beta_O)\beta_I x^{\alpha(1-\eta)-1} - \beta_O(1-\beta_I)(1-x)^{\alpha(1-\eta)-1}\right] \\ \frac{\partial^2 T}{\partial x^2} &= \alpha(1-\eta)\left[\alpha(1-\eta)-1\right]\left[(1-\beta_O)\beta_I x^{\alpha(1-\eta)-2} + \beta_O(1-\beta_I)(1-x)^{\alpha(1-\eta)-2}\right] < 0 \end{aligned}$$

Setting $\frac{\partial T}{\partial x} = 0$, we obtain

$$x^* = \frac{1}{\left[\frac{\beta_O(1-\beta_I)}{\beta_I(1-\beta_O)}\right]^{\frac{1}{1-\alpha(1-\eta)}}+1}$$

When $x > x^*$, $\frac{\partial T}{\partial x} < 0$ holds; when $x < x^*$, $\frac{\partial T}{\partial x} > 0$ follows.

It is easy to show that $x^{SS} = \left[\frac{(1-\beta_O)(1-\beta_I)}{1+\beta_O\beta_I-2\beta_O}\right]^{\frac{1}{1-\alpha(1-\eta)}}$ is a strictly convex function of β_I, *i.e.* $\frac{\partial(x^{SS})}{\partial\beta_I} < 0$, $\frac{\partial^2(x^{SS})}{\partial\beta_I^2} > 0$, and $x^* = \frac{1}{\left[\frac{\beta_O(1-\beta_I)}{\beta_I(1-\beta_O)}\right]^{\frac{1}{1-\alpha(1-\eta)}}+1}$ is a strictly concave function of β_I, *i.e.* $\frac{\partial x^*}{\partial\beta_I} > 0$, $\frac{\partial^2 x^*}{\partial\beta_I^2} < 0$. Meanwhile, when $\beta_I \to \beta_O$, we have $x^{SS} \to 1$, $x^* \to \frac{1}{2}$, and thus $x^{SS} > x^*$;when $\beta_I \to 1$, we obtain $x^{SS} \to 0$, $x^* \to 1$, and hence $x^{SS} < x^*$. So, there exists a unique point $\beta_I^* = f(\beta_O, \alpha, \eta)$ such that when $\beta_I < \beta_I^*$, $x^{SS} > x^*$ applies and when $\beta_I > \beta_I^*$, $x^{SS} < x^*$ follows.

Proof of Proposition 2

It is easy to derive (h^{NS}, m^{NS}) and (h^{SS}, m^{SS}) as

$$\begin{cases} h^{NS} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(T^{NS})^{\frac{\alpha}{1-\alpha}\eta+1}[t(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)}(w^N)^{\frac{-\alpha}{1-\alpha}} \\ m^{NS} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(T^{NS})^{\frac{\alpha}{1-\alpha}\eta}[t(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)+1}(w^N)^{\frac{-\alpha}{1-\alpha}} \end{cases}$$

$$\begin{cases} h^{SS} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}T^{\frac{\alpha}{1-\alpha}\eta+1}[t(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)}(w^N)^{\frac{-\alpha}{1-\alpha}} \\ m^{SS} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}T^{\frac{\alpha}{1-\alpha}\eta}[t(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)+1}(w^N)^{\frac{-\alpha}{1-\alpha}} \end{cases}$$

where

$$\begin{aligned} T^{NS} &= \beta_O\beta_I + (1-\beta_O)\beta_I\left(\frac{1}{B}\right)^{\alpha(1-\eta)} + \beta_O(1-\beta_I)\left(1-\frac{1}{B}\right)^{\alpha(1-\eta)} \\ B &= \left[\frac{t(1-\beta_O)-\beta_O(1-\beta_I)}{(1-\beta_O)(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}} > 1 \\ T &= \beta_O\beta_I + (1-\beta_O)\beta_I\left(\frac{1}{A}\right)^{\alpha(1-\eta)} + \beta_O(1-\beta_I)\left(1-\frac{1}{A}\right)^{\alpha(1-\eta)} \\ A &= \left[\frac{1+\beta_O\beta_I-2\beta_O}{(1-\beta_O)(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}} > 1 \end{aligned}$$

Dividing h^{NS} by h^{SS} and m^{NS} by m^{SS}, we get

$$\frac{h^{NS}}{h^{SS}} = \left(\frac{T^{NS}}{T}\right)^{\frac{\alpha}{1-\alpha}\eta+1} \text{ and } \frac{m^{NS}}{m^{SS}} = \left(\frac{T^{NS}}{T}\right)^{\frac{\alpha}{1-\alpha}\eta}$$

Thus $\frac{h^{NS}}{h^{SS}}$ and $\frac{m^{NS}}{m^{SS}}$ are both increasing in $\frac{T^{NS}}{T}$. For convenience of expression, let G denote $\frac{T^{NS}}{T}$. Then

$$\frac{\partial G}{\partial t} = \frac{G}{T^{NS}}\left(\frac{\partial T^{NS}}{\partial t}\right) = \frac{G}{T^{NS}}\left(\frac{\partial T^{NS}}{\partial B} \times \frac{\partial B}{\partial t}\right)$$

As

$$\frac{\partial B}{\partial t} = \frac{B}{1-\alpha(1-\eta)} \frac{1-\beta_O}{t(1-\beta_O)-(1-\beta_I)\beta_O} > 0$$

we have

$$\frac{\partial G}{\partial t} \sim \frac{\partial T^{NS}}{\partial B} \sim \left[\frac{1}{(B-1)^{1-\alpha(1-\eta)}} - \frac{1}{(x^*-1)^{1-\alpha(1-\eta)}}\right]$$

Let H denote $\frac{1}{(B-1)^{1-\alpha(1-\eta)}} - \frac{1}{(x^*-1)^{1-\alpha(1-\eta)}}$. It is easy to see

$$\frac{\partial H}{\partial t} = -[1-\alpha(1-\eta)](B-1)^{\alpha(1-\eta)-2}\frac{\partial B}{\partial t} < 0$$

Thus $\frac{1}{(B-1)^{1-\alpha(1-\eta)}} - \frac{1}{(x^*-1)^{1-\alpha(1-\eta)}}$ is decreasing in t. When $t \to 1$, $B \to A < x^*$, then $\frac{1}{(B-1)^{1-\alpha(1-\eta)}} - \frac{1}{(x^*-1)^{1-\alpha(1-\eta)}} > 0$ and as a result, $\frac{\partial G}{\partial t} > 0$; when $t \to \infty$, $B \to \infty$, then $\frac{1}{(B-1)^{1-\alpha(1-\eta)}} - \frac{1}{(x^*-1)^{1-\alpha(1-\eta)}} < 0$ and as a result, $\frac{\partial G}{\partial t} < 0$. We therefore conclude that G is first increasing and then decreasing in t.

Since $G(t=1) = 1$ and $G(t=\infty) = \frac{\beta_O}{T} < 1$, we know that $G > 1$ and thus $(h^{NS}, m^{NS}) > (h^{SS}, m^{SS})$ hold when t is not so large and $G < 1$ and thus $(h^{NS}, m^{NS}) < (h^{SS}, m^{SS})$ hold when t becomes sufficiently large. Given the complementarity between h and m in the Cobb-Douglas production function (3), we know that NS achieves a higher level of production efficiency than SS.

Proof of Proposition 3

We also can get the optimal (h^{SN}, m^{SN}) from equation (11)

$$\begin{cases} h^{SN} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(T^{SN})^{\frac{\alpha}{1-\alpha}\eta+1}[(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)}(w^N)^{\frac{-\alpha}{1-\alpha}} \\ m^{SN} = \eta\alpha^{\frac{1}{1-\alpha}}(\mu I)^{\frac{1}{1-\alpha}}Y^{-\frac{\alpha}{1-\alpha}}\theta^{\frac{\alpha}{1-\alpha}}(T^{SN})^{\frac{\alpha}{1-\alpha}\eta}[(1-\beta_O)]^{\frac{\alpha}{1-\alpha}(1-\eta)+1}(w^N)^{\frac{-\alpha}{1-\alpha}} \end{cases}$$

where

$$\begin{aligned} T^{SN} &= \beta_O\beta_I + (1-\beta_O)\beta_I\left(\frac{1}{C}\right)^{\alpha(1-\eta)} + \beta_O(1-\beta_I)\left(1-\frac{1}{C}\right)^{\alpha(1-\eta)} \\ C &= \left[\frac{(1-\beta_O)-t\beta_O(1-\beta_I)}{t(1-\beta_O)(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}} > 1 \\ 1 &\leq t \leq \frac{1-\beta_O}{1-\beta_I} \end{aligned}$$

Dividing h^{SN} by h^{SS} and m^{SN} by m^{SS}, we get

$$\frac{h^{SN}}{h^{SS}} = \left(\frac{T^{SN}}{T}\right)^{\frac{\alpha}{1-\alpha}\eta+1}\left(\frac{1}{t}\right)^{\frac{\alpha}{1-\alpha}(1-\eta)}, \text{ and } \frac{m^{SN}}{m^{SS}} = \left(\frac{T^{NS}}{T}\right)^{\frac{\alpha}{1-\alpha}\eta}\left(\frac{1}{t}\right)^{\frac{\alpha}{1-\alpha}(1-\eta)+1}$$

For convenience of expression, let $u = \frac{1}{t}$ and $N = T(x(u))^{\frac{\alpha}{1-\alpha}\eta+1} u^{\frac{\alpha}{1-\alpha}(1-\eta)}$, where $T(x) = \beta_O\beta_I + (1-\beta_O)\beta_I x^{\alpha(1-\eta)} + (1-\beta_I)\beta_O(1-x)^{\alpha(1-\eta)}$, $x(u) = \left[\frac{(1-\beta_O)(1-\beta_I)}{u(1-\beta_O)-\beta_O(1-\beta_I)}\right]^{\frac{1}{1-\alpha(1-\eta)}}$ and u lies in the interval of $[\frac{1-\beta_I}{1-\beta_O}, 1]$. Taking logarithm of N, we have

$$\ln N = \left(\frac{\alpha}{1-\alpha}\eta + 1\right)\ln T(x) + \frac{\alpha}{1-\alpha}(1-\eta)\ln u$$

Thus

$$\frac{\partial N}{\partial u} = N\left[\left(\frac{\alpha}{1-\alpha}\eta+1\right)\frac{1}{T}\frac{\partial T}{\partial x}\frac{\partial x}{\partial u} + \frac{\alpha}{1-\alpha}(1-\eta)\frac{1}{u}\right]$$

It has two parts, the second of which is definitely positive. We need to determine the sign of the first part. As

$$\frac{\partial x}{\partial u} = \frac{-x}{1-\alpha(1-\eta)}\left[\frac{(1-\beta_O)}{(1-\beta_O)u-(1-\beta_I)\beta_O}\right] < 0$$

and

$$\frac{\partial T}{\partial x} = \alpha(1-\eta)\left[(1-\beta_O)\beta_I x^{\alpha(1-\eta)-1} - \beta_O(1-\beta_I)(1-x)^{\alpha(1-\eta)-1}\right]$$

The sign of $\frac{\partial N}{\partial u}$ depends on that of $\frac{\partial T}{\partial x}$. When $\frac{\partial T}{\partial x} < 0$, which happens if u is low and $x < x^*$, we have $\frac{\partial N}{\partial u} > 0$. In contrast, when $\frac{\partial T}{\partial x} > 0$, which occurs if u is high and $x > x^*$, the sign of $\frac{\partial N}{\partial u}$ is ambiguous. In the following we consider this latter case, *i.e.* $x > x^*$ and $\frac{\partial T}{\partial x} > 0$.

Rewrite $\frac{\partial N}{\partial u}$ as

$$\frac{\partial N}{\partial u} = \frac{N}{u}\left[\frac{\alpha}{1-\alpha}(1-\eta) + \left(\frac{\alpha}{1-\alpha}\eta+1\right)g(u)\right]$$

where $g(u) = \frac{u}{T}\frac{\partial T}{\partial x}\frac{\partial x}{\partial u}$

$$\frac{\partial g(u)}{\partial u} = \frac{1}{T}\frac{\partial T}{\partial x}\frac{\partial x}{\partial u} - \frac{u}{T^2}\left(\frac{\partial T}{\partial x}\frac{\partial x}{\partial u}\right)^2 + \frac{u}{T}\frac{\partial^2 T}{\partial x^2}\left(\frac{\partial x}{\partial u}\right)^2 + \frac{u}{T}\frac{\partial T}{\partial x}\frac{\partial^2 x}{\partial u^2} < 0$$

as $\frac{\partial^2 T}{\partial x^2} < 0$, $\frac{\partial T}{\partial x} > 0$, $\frac{\partial^2 x}{\partial u^2} > 0$. So

$$\begin{aligned}\frac{\partial N}{\partial u} &\geq N\left[\frac{\alpha}{1-\alpha}(1-\eta) + \left(\frac{\alpha}{1-\alpha}\eta+1\right)g(u=1)\right] \\ &= N\frac{\alpha}{1-\alpha}(1-\eta)[1- \\ &\quad \frac{1-\beta_O}{1-\beta_O-\beta_O(1-\beta_I)}\frac{\beta_I(1-\beta_O)-\beta_O(1-\beta_I)(A-1)^{\alpha(1-\eta)-1}}{\beta_O\beta_I A^{\alpha(1-\eta)} + (1-\beta_O)\beta_I + (1-\beta_I)\beta_O(A-1)^{\alpha(1-\eta)}}]\end{aligned}$$

It is easy to see that the expression on the right hand side is bigger than zero. So $\frac{\partial N}{\partial u} > 0$ holds.

Now we know that N is always increasing in u. When u reaches its maximum, i.e., $u = 1$, N also reaches the maximum, i.e., $N_{\max} = T^{\frac{\alpha}{1-\alpha}\eta+1}$. Under this circumstance, (h^{SN}, m^{SN}) and (h^{SS}, m^{SS}) are equal. This implies that when $u < 1$, $N < N_{\max}$ and thus $(h^{SN}, m^{SN}) \leq (h^{SS}, m^{SS})$ holds.