Excludable Public Goods: Pricing and Social Welfare Maximization

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Abstract

We compare two pricing strategies – buffet pricing and usage pricing – of excludable public goods for social welfare maximization. Buffet pricing is better than usage pricing for low consumer heterogeneity, while the opposite holds for high consumer heterogeneity.

Key Words: Excludable Public Goods, Per-unit Usage Pricing, Buffet Pricing, Social Welfare Maximization
JEL Codes: H41, D40

1 Introduction

The literature on excludable public goods has focused on the provision of those goods either by public agencies, or private firms, or public-private partnerships, and concerned about the issues of information asymmetries and moral hazard involved.[1] It is arguable, however, that the ultimate objective for having those public goods is to maximize social welfare. The realization of social welfare with excludable public goods, in turn, depends on the specific pricing strategies used for the access to these goods. The case

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[1]See, for example, Breithut (1939), Vickrey (1963, 1969), Winston (1991), Lee (1991), Oum and Zhang (1990), Fraser (1996), and Cremer and Laffont (2003).

of the Japanese national highway system provides an excellent example of how poorly designed pricing strategy could lead to substantial losses in social welfare. As reported by Jason Singer (2003) in the *Asian Wall Street Journal* on September 15, 2003 that, due to hefty toll fees, Japanese drivers tried everything possible to avoid driving on the National highways, leaving the National highways empty but local routes congested. Meanwhile, the City of Chongqing in China has witnessed a significant increase in both car registration and usage since it changed its road charges from toll fees to annual passes on July 1, 2002.

Despite the importance of pricing strategies for excludable public goods and their impacts on social welfare, there is limited work on this topic. This paper fills in the void by assuming away the issue of provision and focusing instead on the comparison of various pricing strategies in terms of social welfare generated. There are two commonly used pricing strategies for excludable public goods: per-unit *usage pricing*, and *buffet pricing* where consumers can enjoy any amount of excludable public goods for a certain period of time once paying a lump-sum fee in advance. Using a model where consumers differ in their willingness to pay, we find that buffet pricing gives higher social welfare than usage pricing for the case of low consumer heterogeneity. For a uniform distribution of consumer's willingness to pay, we further find that while buffet pricing is still preferred to usage pricing for the case of low consumer heterogeneity, the opposite holds for the case of high consumer heterogeneity. We also extend our analysis by investigating the conditions under which simultaneous use of usage pricing and buffet pricing gives the highest social welfare.

A paper related to ours is by Nahata, Ostaszewski and Sahoo (1999).[2] They compare buffet pricing with usage pricing in terms of profit generated under the assumption that usage pricing involves an extra marginal cost than buffet pricing does. In contrast, we do not assume an extra marginal cost associated with usage pricing and furthermore our focus is on the comparison of these two pricing strategies in terms of social welfare generated.

The paper is organized as follows. The model setup for the analysis is laid out in Section 2, and the main analysis is presented in Section 3. The paper concludes with Section 4.

[2]Other related studies include the pricing of public intermediate goods (e.g., Feldstein (1971) and, Yang (1991)), and the pricing of shared facilities (e.g., Scotchmer 1985).

2 Model Setup

A government agency considers building an infrastructural facility (for example, highways, museums, and parks) at a fixed cost I. Once constructed, the facility provides an excludable-public good G to a community of consumers at zero marginal cost. Consumer demand for G is given by

$$q_i(p) = \theta_i - p, \tag{1}$$

where q_i is the quantity of consumption and θ_i represents consumer i's highest willingness to pay. θ_i is private information, and the government agency only knows its cumulative distribution function $F(\theta)$ and the density distribution function $f(\theta)$. θ_i is assumed to lie in the support of $[\theta_0 - \varepsilon, \theta_0 + \varepsilon]$, where $\varepsilon \in [0, \theta_0]$ represents the degree of consumer heterogeneity.

Consider two commonly-used pricing strategies that the government agency can use for the excludable public good G. One is the per-unit *usage pricing*. Given the usage price p, only those consumers whose willingness to pay is higher than the price (i.e., $\theta_i > p$) will choose to enjoy good G. Let $\Omega_p(\varepsilon)$ denote the set of participating consumers under usage pricing. The associated revenue and the social welfare are

$$\begin{cases} \pi_p = \int_{\theta \in \Omega_p(\varepsilon)} p q_i(p) f(\theta) d\theta \\ SW_p = \int_{\theta \in \Omega_p(\varepsilon)} \left[\left(\int_p^{\theta} q_i(t) dt \right) + p q_i(p) \right] f(\theta) d\theta \end{cases} \tag{2}$$

The other pricing strategy is *buffet pricing* where consumers can enjoy any amount of G once paying a lump-sum fee T in advance. It can be shown that only those consumers with $\frac{\theta_i^2}{2} > T$ will choose to enjoy good G. Let $\Omega_T(\varepsilon)$ denote the set of participating consumers under buffet pricing. The associated revenue and the social welfare are

$$\begin{cases} \pi_T = \int_{\theta \in \Omega_T(\varepsilon)} T f(\theta) d\theta \\ SW_T = \int_{\theta \in \Omega_T(\varepsilon)} \left(\int_0^{\theta} q_i(t) dt \right) f(\theta) d\theta \end{cases} \tag{3}$$

When consumers are homogeneous ($\varepsilon = 0$), the highest buffet price chargeable is $\frac{\theta_0^2}{2}$ and the highest usage price chargeable is θ_0. It can be shown that the maximum revenue under optimal buffet pricing is $\frac{\theta_0^2}{2}$ whereas that under usage pricing is $\frac{\theta_0^2}{4}$. To make the analysis non-trivial, it is assumed that the investment cost can be recovered under this homogenous case, i.e., $I < \frac{\theta_0^2}{4}$.

The government agency's objective is to maximize social welfare subject to the constraint that the fixed cost I can be recovered (*investment recovery constraint*).

$$\max_{j} \quad SW_j$$
$$s.t.\ \pi_j \geq I \tag{4}$$

where $j \in \{T, p\}$. The government agency will choose the pricing strategy that gives a higher social welfare.

3 Usage Pricing versus Buffet Pricing

In comparing the social welfare achieved under usage pricing versus that under buffet pricing, it is useful to introduce the concept of *full-participation*, i.e., all consumers can enjoy good G while the fixed cost of investment can be recovered. In the presence of consumer heterogeneity, the maximum buffet price should be less than $\frac{(\theta_0-\varepsilon)^2}{2}$ in order to have all consumers enjoy the good G. It can be shown that, under this case, the investment cost I can only be recovered when the degree of consumer heterogeneity is low enough (i.e., $\varepsilon < \varepsilon_T$). Similarly, the maximum usage price for ensuring all consumers to enjoy the good G is $\theta_0 - \varepsilon$; and the investment cost I can be recovered if $\varepsilon < \varepsilon_p$.

Define the *case of low consumer heterogeneity* as that where there is full participation under both pricing strategies, i.e., $\varepsilon \leq \min\{\varepsilon_T, \varepsilon_p\}$, and the *case of high consumer heterogeneity* as that where full participation is not satisfied in either pricing strategy, i.e., $\varepsilon > \max\{\varepsilon_T, \varepsilon_p\}$. We have:[3]

Proposition 1 *(i) For the case of low consumer heterogeneity , social welfare under optimal buffet pricing is higher than that under optimal usage pricing. (ii) For the case of high consumer heterogeneity , social welfare under optimal buffet pricing is higher than that under optimal usage pricing when the following condition holds:*

$$\int_{p^*}^{\theta_0+\varepsilon} \left(\int_0^{p^*} q_i(t)dt - p^* q_i(p^*) \right) f(\theta)d\theta > \int_{p^*}^{m^*} \left(\int_0^{\theta} q_i(t)dt \right) f(\theta)d\theta \tag{5}$$

where p^* is the optimal usage price and $m^* = \sqrt{2T^*}$is a transform of the optimal buffet price T^*.

The intuition for the results is as follows. There are two sources of loss in social welfare under either of these two pricing strategies. One is the loss

[3]All proofs are contained in the Appendix, which is available upon request.

occurred when not all consumers can enjoy the good G (called *Participation Loss*), and the other is the loss when the consumption level of a consumer is below the level at zero usage price (called *Consumption Loss*).

For the case of low consumer heterogeneity, there is no participation loss under either of the two pricing strategies. As for consumption loss, however, there exists under optimal usage pricing but not under optimal buffet pricing. Hence the result of Proposition 1 (i). For the case of high heterogeneity, there are both participation loss and consumption loss under optimal usage pricing, but there is only participation loss under optimal buffet pricing. Which one has higher social welfare hinges upon condition (5), where the left side is the consumption loss under optimal usage pricing and the right side is the difference in the participation loss between optimal buffet pricing and optimal usage pricing.

Next, using a uniform distribution of consumer's maximum willingness to pay, $f(\theta) = 1/2\varepsilon$, we can characterize the optimal pricing strategy for all types of consumer heterogeneity:

Proposition 2 *Under the uniform distribution of consumer's maximum willingness to pay, there exists a degree of consumer heterogeneity,* $\varepsilon \in (\min\{\varepsilon_T, \varepsilon_p\}, \max\{\varepsilon_T, \varepsilon_p\})$ *below which social welfare under optimal buffet pricing is higher than that under optimal usage pricing, but above which the opposite holds.*

In real-world situation, we observe the use of usage pricing and buffet pricing simultaneously. For example, parks often offer monthly passes as well as per entrance fees. Now we consider the choice of either pure usage pricing, or pure buffet pricing, or both, and find that simultaneous use of both pricing strategies could be optimal in some cases.

Proposition 3 *Under the uniform distribution of consumer's maximum willingness to pay, social welfare under optimal buffet pricing is the highest for the case of low consumer heterogeneity, but the simultaneous use of buffet pricing and usage pricing generates the highest social welfare for the case of high consumer heterogeneity.*

4 Conclusion

In this paper, we compare two commonly used pricing strategies – usage pricing and buffet pricing – of excludable public goods in terms of social welfare maximization. Using a model where consumers differ in their willingness to pay, we find that buffet pricing gives higher social welfare than usage pricing

for the case of low consumer heterogeneity. For uniform distribution of consumer's willingness to pay, we further find that while buffet pricing is still preferred to usage pricing for the case of low consumer heterogeneity, the opposite holds for the case of high consumer heterogeneity. We also extend our analysis by investigating the conditions under which simultaneous use of usage pricing and buffet pricing gives the highest social welfare.

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